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THEORY OF GROUNDWATER FLOW IN THE VICINITY OF BRINE PONDS AND SALT TAILINGS PILES

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1. INTRODUCTION

This report reviews some of the groundwater flow theory which can be used to predict and analyse the impact of brine ponds and salt tailings piles on groundwater flow. It is based on a hydrogeological setting typical for southern Saskatchewan.

All the potash mining operations in southern Saskatchewan have large waste disposal basins. Generally such a basin consists of a brine pond and a tailings pile. Because a pond may have a different impact on groundwater flow than a tailings pile, the influence of a pond and of a pile are discussed separately. Also, because the water in a brine pond has a density greater than one, some aspects of flow of fluids with different densities are discussed.

This report does not purport to be complete in its treatment of the potential impact of a waste disposal basin on groundwater flow. Most of the theories discussed can be found in published literature. At all times it should be realized that the theories are based on assumptions simplifying the actual hydrogeological setting. Complex computer programs and large amounts of data would be required to simulate actual conditions. However, provided good judgement is used, the theories discussed in this report can be used in predicting and analyzing the impact of a waste disposal basin on groundwater flow.

2. SCHEMATIC HYDROGEOLOGICAL SETTING

In Figure 1, a hydrogeological setting typical for Saskatchewan is illustrated schematically. This four-layer setting includes, in descending order; an unconfined surficial aquifer, a semi-confining layer or aquitard, a semi-confined aquifer and a basal aquitard. The surficial aquifer can be thought to consist of a sand or of a fractured till. Its important hydraulic characteristic is that its hydraulic or bulk hydraulic conductivity is several orders of magnitude larger than the hydraulic conductivity of the aquitard below it. A semi-confining layer or aquitard is defined as a layer which has a much lower hydraulic conductivity than the aquifers above or below it. Flow in an aquitard is vertically upward or downward as the horizontal flow component can be neglected. The semi-confined aquifer is bounded below by a basal aquitard.

The waste disposal basin, assumed to consist of a pond and a tailings pile, is situated on top of the surficial aquifer (see Figure 1).

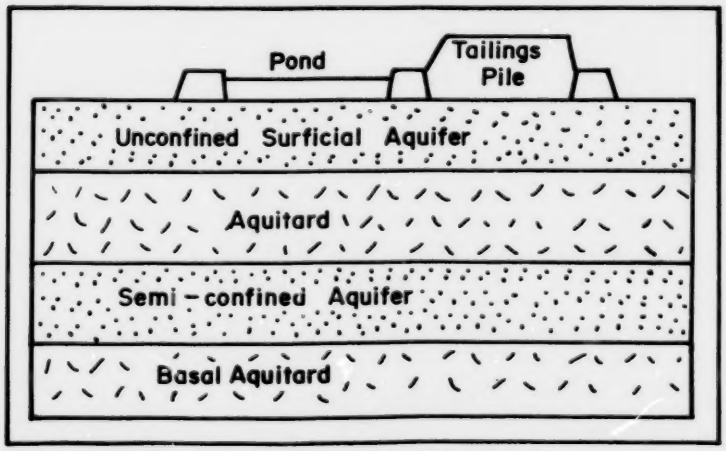

Fig. 1 Schematic hydrogeological setting.

3. SOME NOTES ON VERTICAL HYDRAULIC RESISTANCE AND LEAKAGE FACTOR

In relation to semi-confined aquifers, the vertical hydraulic resistance of the overlying aquitard and a parameter called leakage factor are fundamental geohydrological constants.

The vertical hydraulic resistance, of an aquitard, is defined as the ratio between the saturated thickness of the aquitard and is vertical hydraulic conductivity. In equation form, the vertical hydraulic resistance is expressed as:

$$c = \frac{b'}{K'_v} \qquad (3\text{-}1)$$

where: c = vertical hydraulic resistance

b = thickness of aquitard

K_v= vertical hydraulic conductivity of aquitard.

The vertical hydraulic resistance c of an aquitard characterizes the resistance of the aquitard to upward or downward flow. For vertical flow, Darcy's Law can be expressed as:

$$q = \frac{\Delta h}{c} \qquad (3\text{-}2)$$

where: Δh = head difference across aquitard.

Equation 3-2, for instance, indicates that if c is large, the amount of upward or downward flow will be small.

The leakage factor L is defined as:

$$L = \left(\frac{K\ b\ b'}{K'_v}\right)^{1/2} = (Tc)^{1/2} \qquad (3\text{-}3)$$

where: L = leakage factor

K = hydraulic conductivity of semi-confined aquifer

b = thickness of semi-confined aquifer

T = Kb = transmissivity of semi-confined aquifer.

As an example of the meaning of the leakage factor, consider steady-state flow and a well in an infinite, uniform, semi-confined aquifer. The drawdown in the aquifer is given by (deGlee, 1930; Hantush and Jacob, 1955):

$$s = \frac{Q}{2\pi T} K_o(r/L) \qquad (3\text{-}4)$$

where: s = drawdown in aquifer

Q = pumping rate

T = transmissivity of semi-confined aquifer

$K_o(x)$ = Bessel function of order zero and second kind

r = distance from well

L = leakage factor

Consider for instance a well with a radius of 0.10 m completed in a semi-confined aquifer with an L-value of 1000 m. If the steady drawdown at the well is 10 metres then the drawdowns at 100 m (r/L = 0.1), 1000 m (r/L = 1.0) and 2000 m (r/L = 2.0) are 2.60 m, 0.45 m and 0.12 m respectively.

Knowledge of L quickly gives a good impression of the size of the area most effected by pumpage or injection. In Saskatchewan the L value of major intertill and buried valley aquifers typically may range from

hundreds of metres to several tens of kilometres (see for example, van der Kamp et al., 1986).

The importance and significance of the leakage factor L is emphasized throughout this report, in particular in the sections dealing with analysis of water levels and hydrodynamic containment in semi-confined aquifers.

If the value of c in a particular case is estimated to be less than about 10,000 days, it may be possible to determine a reliable value for c and L by means of a pumping test of sufficient duration (see for example Kruseman and de Ridder, 1970; van der Kamp, 1985). If the value of c is likely to exceed 10,000 days a pumping test determination may not be practical and other methods for determining K'_v, c and L are indicated. Some methods for determining K'_v, c and L which use the effects of waste disposal basin on water levels are discussed in this report.

4. THEORY OF FLOW OF FLUIDS WITH DIFFERENT DENSITY

4.1 Hydraulic Conductivity for Different Fluids

Fluid density differences within an aquifer or aquifer system may have an important effect on the fluid movement. Subsurface brine migration may be strongly influenced by density effects.

The flow of any fluid with constant density can be calculated by means of Darcy's law:

$$q = K\,\mathrm{grad}(h) \tag{4-1}$$

where: q = specific discharge (vector)

K = hydraulic conductivity

h = fluid head

The hydraulic conductivity K is related to the density ρ and dynamic viscosity μ of the fluid by:

$$K = \frac{\rho g k}{\mu} \tag{4-2}$$

where: k = intrinsic permeability

ρ = fluid density

g = acceleration of gravity

μ = dynamic viscosity

Note that the hydraulic conductivity varies with density and is also sensitive to temperature since the dynamic viscosity is strongly dependent on temperature. However, if the temperature remains constant, the dynamic viscosity can be considered independent of the salt content of the fluid (U.S. National Research Council, 1933). The hydraulic conductivities for fresh and saline water are then related according to (Ten Hoorn, 1980):

$$K_s = \frac{\rho_s}{\rho_f} K_f \tag{4-3}$$

where: K_s = hydraulic conductivity for saline water

K_f = hydraulic conductivity for fresh water

ρ_s = density of saline water

ρ_f = density of fresh water

In addition, depending on the chemical composition of the fluid, interactions between clay minerals and the fluid may have a significant effect on the hydraulic conductivity. Goldenberg et al (1983) reported on virtually irreversible decreases in hydraulic conductivity when fresh water replaced sea water in column experiments. Davis and DeWiest (1966) suggest that due to chemical interactions in clay-rich sediments the brine hydraulic conductivity may be 100 times larger than the hydraulic conductivity for distilled water. Ho and Pufahl (1986) present detailed results for the effects of brine contamination on the properties of soils.

4.2 Fresh-Water Head and Saline-Water Head

The fluid head or hydraulic head h is defined by:

$$h = \frac{p}{\rho g} + z \qquad (4\text{-}4)$$

where: h = fluid head or hydraulic head

p = pressure

z = elevation

The important point with respect to saline fluid migration is that the flow of saline water through the subsurface depends on what may be called the saline water head, h_s;

$$h_s = \frac{p}{\rho_s g} + z \qquad (4\text{-}5)$$

where: ρ_s is the density of the saline fluid.

On the other hand, the flow of fresh water is governed by the fresh water head, h_f;

$$h_f = \frac{p}{\rho_f g} + z \qquad (4\text{-}6)$$

where ρ_f is the density of fresh water ($\rho_f = 1000$ kg/m^3). It is not possible to describe the flows of both saline water and of fresh water in terms of one variable such as fresh-water head. For movement of fresh water, fresh-water head must be used; for movement of saline water, saline-water head must be used.

It should be realized that, whether or not the fluid at a particular point is saline or fresh, at any point within a porous medium equations 4-6 and 4-5 can be used to calculate the hydraulic head either in terms of fresh-water head or saline-water head. The relationship between the two heads is:

$$h_s = \frac{\rho_f}{\rho_s} h_f + (1 - \frac{\rho_f}{\rho_s}) z \qquad (4\text{-}7)$$

$$h_f = \frac{\rho_s}{\rho_f} h_s + (1 - \frac{\rho_s}{\rho_f}) z \qquad (4\text{-}8)$$

In the case where the saline fluid is saturated brine ($\rho_b = 1230$ kg/m^3), the relationship between the heads becomes:

$$h_b = 0.813\, h_f + 0.187\, z \qquad (4\text{-}9)$$

$$h_f = 1.23\, h_b - 0.23\, z \qquad (4\text{-}10)$$

where: h_b = brine head.

Generally water-levels and gradients are expressed in terms of fresh-water heads. However, it is important to note that the difference in saline-water head between two points is not simply proportional to the difference in fresh-water head between these points. To illustrate

this point, assume the saline fluid is brine; the difference in brine head then can be expressed as:

$$\Delta h_b = 0.813\ \Delta h_f + 0.187\ \Delta z \qquad (4\text{-}11)$$

If Z equals zero (horizontal flow), equation 4-10 becomes

$$\Delta h_b = 0.813\ \Delta h_f \qquad (4\text{-}12)$$

and indicates that the brine gradient is 0.813 times the fresh-water gradient. If ΔZ is not equal to zero (vertical flow), equation 4-11 has to be applied.

Another interesting case develops if the fresh-water head difference is zero. Equation 4-11 then becomes:

$$\Delta h_b = 0.187\ \Delta z \qquad (4\text{-}13)$$

If the reference level is taken as zero metres ASL, equation 4-13 shows that with increasing elevation above the reference level the brine head increases though the fresh-water head is constant. Consequently, as flow of brine is from a high brine head to a low brine head, equation 4-13 as an exceptional case and equation 4-11 as the general case, show the tendency of brine to move downward.

4.3 Lateral Migration of Saline Fluids

Lateral migration of a saline fluid in a non-saline environment may occur in both an unconfined surficial aquifer and in a semi-confined aquifer.

In both cases the following should be remembered:

- Where the pore fluid in a formation consists entirely of a saline fluid with constant density, its movement can be described with Darcy's Law, using the gradient of the saline-water head.

- Where both fresh water and saline water are present in the formation, the fresh water movement is governed by the gradient of the fresh-water head while the direction and rate of the movement of saline water is governed by the gradient of the saline-water head.

In Figure 2 the simple case of steady flow or a "dynamic equilibrium" between fresh water and saline water in an unconfined surficial aquifer is portrayed. With steady flow is meant that the interface between the fresh water and saline water is not moving. In order to maintain a stationary interface, the fresh water is flowing towards the ditch. Under these simplified conditions, the Ghyben-Herzberg relation can be applied to all points along the interface (Todd, 1959). The Ghyben-Herzberg relation can be expressed as:

$$Z_w = \frac{\rho_s - \rho_f}{\rho_f} \cdot Z_B \qquad (4\text{-}14)$$

where: Z_w = height of water table above the bottom of the ditch

Z_B = depth to interface

If the water table configuration is known, equation 4-14 could be used to estimate the position of the interface relative to the bottom of the ditch. At the bottom of the aquifer the saline fluid will penetrate to the point where:

$$Z_w = \frac{\rho_s - \rho_f}{\rho_f} \cdot D \qquad (4\text{-}15)$$

where: D = thickness of surficial aquifer below the bottom of the ditch

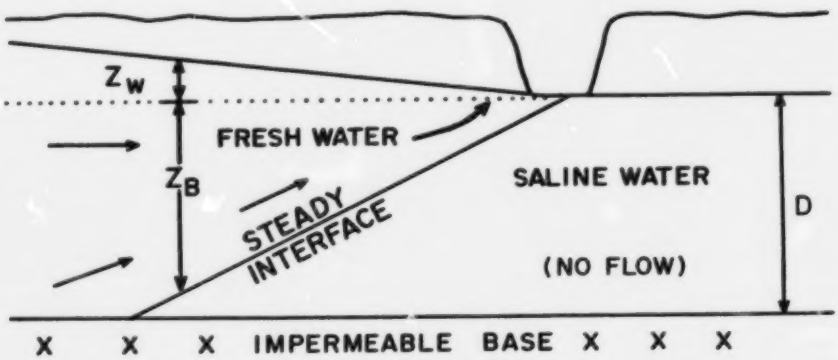

Fig. 2 Sketch of the interface between fresh and saline water: lateral migration.

The distance to which the saline fluid will migrate is difficult to predict. It depends on the original water table configuration, the depth of the ditch and the influence of the ditch on the water table configuration. In general, however, the deeper the ditch the shorter the distance will be from the ditch to the point where migration is halted.

The case discussed above is based on the assumptions that the aquifer is homogeneous and isotropic, and that the interface is sharp. In actual practice the interface is not sharp but exists as a wide mixing zone, due to diffusion and dispersion. In some instances, the processes of diffusion and dispersion may be extremely effective and the zone of mixing can be hundreds of metres wide. This situation may be particularly true for strongly heterogeneous formations. For such cases, the first appearance of slightly saline water may occur at points far beyond the saline water itself, and it may be possible that the saline water may never penetrate out as far as these points. In addition, the discharging fresh water tends to entrain some saline water from the mixing zone.

The net result of flow between two fluids with different densities generally is a complex pattern of flow and salinity in the zone where these fluids meet. The problem of describing and predicting these patterns can only be handled by means of complex computer models (e.g. Frind, 1982). However, using the equations derived in Section 4-2, it is possible to estimate the overall tendency of lateral movement of the saline water front by calculating within the adjoining zones the rate of movement of fresh water at one side and of saline water at the other side, using gradients of fresh-water head and saline-water head, respectively.

4.4 Vertical Migration of Saline Fluids

Density effects may also play an important role in the vertical migration of a saline fluid. The extra density of the saline fluid causes an increased tendency for downward movement.

Figure 3 shows a pond containing saline water situated on top of an aquitard. It is assumed that the saline-water front has moved down into the formation below the pond to an elevation Z_i. If the hydraulic conductivity does not change, the rate of downward movement of the saline-water front is given by:

$$v = \frac{K_v'}{n} \left[\frac{(Z_p - h_a) + \frac{\rho s - \rho f}{\rho_f} (Z_p - Z_i)}{b'} \right] \quad (4\text{-}16)$$

where: v = rate of movement of saline-water front

K_v' = vertical hydraulic conductivity of the aquitard

n = effective porosity of the aquitard

Z_p = brine pond elevation

h_a = fresh-water head in aquifer

Z_i = elevation of fresh-brine interface

b' = thickness of aquitard

ρ_s = density saline water

ρ_f = density fresh water

Equation 4-16 shows that the total driving head is equal to the water-level difference Z_p-h_a plus the extra head due to the extra weight of the total saline-water column. As the interface moves down, the length of the saline-water column increases and consequently, the rate of downward movement increases too.

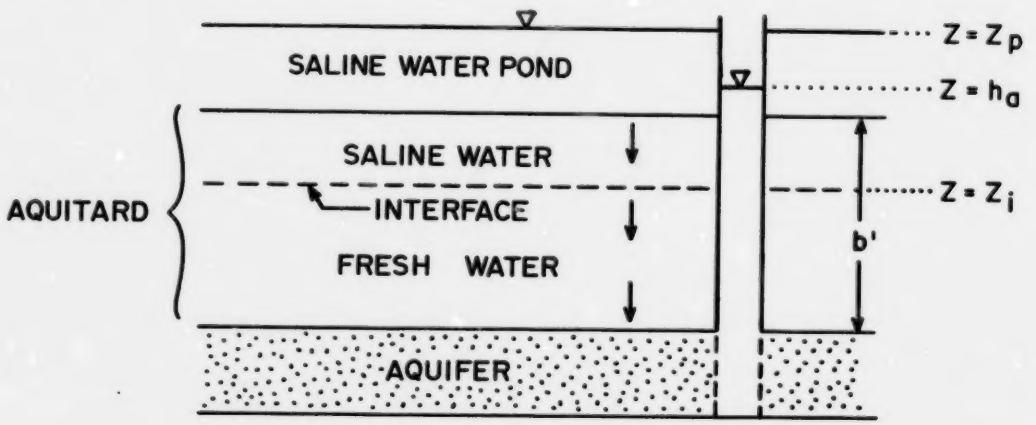

Fig. 3 Sketch of the interface between fresh and saline water: vertical migration.

It should be remembered that due to diffusion and dispersion the interface is not sharp but occurs as a zone of mixing. In particular, if the formation is fractured, the shape of the interface may be highly irregular. In such fractured formations, it is possible for the saline fluid to have reached the underlying aquifer through fracture flow while it still may take a very long time for the saline-water front in the matrix to reach the aquifer.

The tendency of the heavier saline fluid to move downward can lead to apparently "up-gradient" flow. If the base of the aquifer is not horizontal, the saline fluid will tend to move down the slope along the base of the aquifer, even if the fresh-water hydraulic gradient is in another direction. Such cases can be analyzed or predicted by means of equations 4-7 through 4-10. Equation 4-9 for instance shows that the gradient of brine head has a slope-related component equal to 0.187 times the dip of the slope.

In general saline fluid at the base of an aquifer will tend to flow in another direction than the fresh water above it. Depending on the topography of the base of the aquifer, this effect can lead to "tongues" of saline water moving in unexpected directions.

5. ANALYSIS AND PREDICTION OF WATER LEVELS IN A SEMI-CONFINED AQUIFER BENEATH AND AROUND A WASTE DISPOSAL POND

5.1 Introduction

Of particular interest is the influence of a waste disposal basin on the water levels in a semi-confined aquifer beneath and around the basin. Based on the ratio between the radius of the basin R and leakage factor L, two models can be distinguished for the purpose of analysis and prediction:

1. If the ratio R/L is large, the basin can be considered to consist of straight dykes which separate the pond from its surroundings.
2. If the ratio R/L is small, the basin can be considered as a circle.

While a mathematical derivitation is not provided, Verruijt (1970) suggests that if R/L is greater than 2.5, the basin can be considered as consisting of straight dykes.

The two models will be discussed further in the next sections.

5.2 Straight Dyke Model

In Figure 4 the straight dyke model is illustrated schematically. On the right side of the dyke water is moving vertically downward from the basin into the underlying semi-confined aquifer: on the left side, water is moving from the aquifer toward the water table.

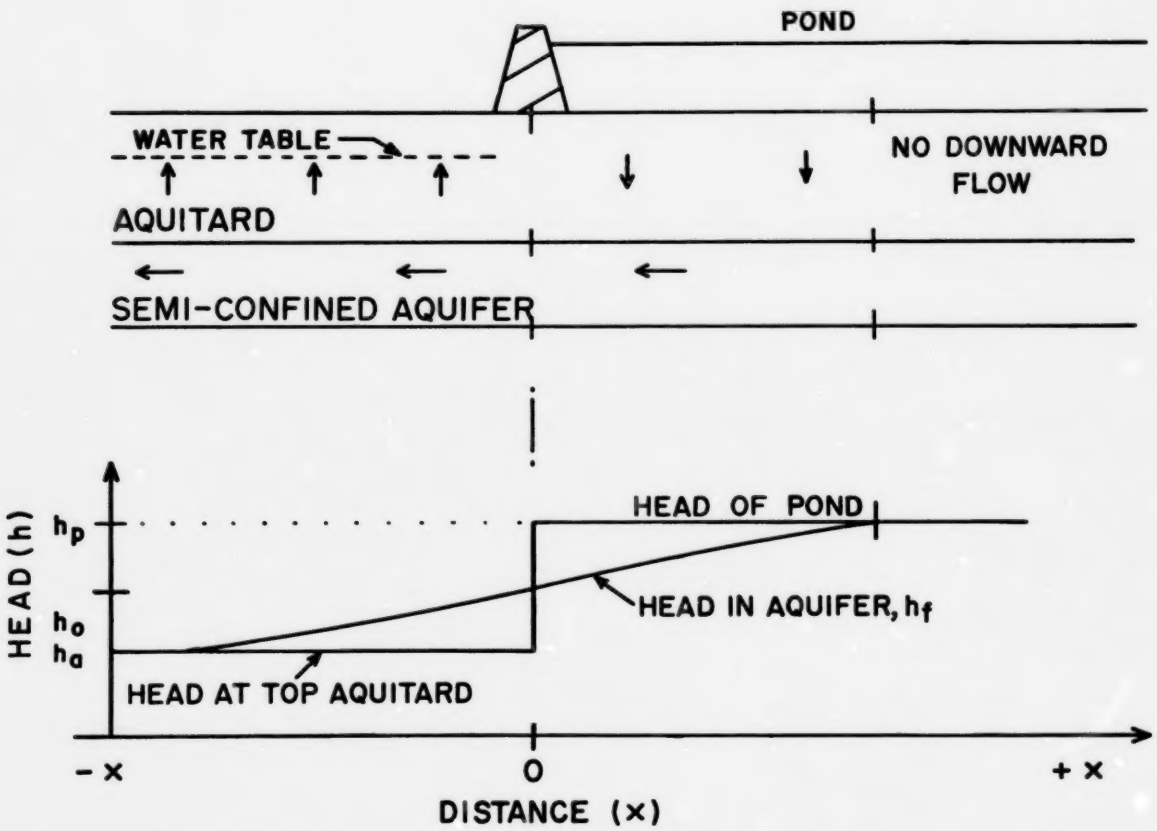

Fig. 4 Schematic illustration of heads across a straight dyke separating a pond from its surroundings.

The following assumptions are made:

1. The ratio R/L is greater than 2.5.
2. The hydrogeological setting does not change for a distance of at least 2.5 L around the basin.
3. The pond rests on an aquitard. In the hydrogeological model used in this report, the aquitard is overlain by a surficial aquifer. By combining the vertical hydraulic resistance of the surficial aquifer and the aquitard, these two layers can be represented as one aquitard.
4. The width of the dyke is very small compared to the dimensions of the pond and the L value.
5. Outside the dyke the water level in the aquitard is horizontal and constant.

The basic differential equation governing this case of groundwater flow is given by Verruijt (1970):

$$\frac{\partial^2 h_f}{\partial x_2} = \frac{h_f - h_a}{L^2} \qquad (5\text{-}1)$$

where: h_f = head in semi-confined aquifer

h_a = head at top of aquitard

L = leakage factor [$L = (Tc)^{1/2}$]

The boundary conditions for the conditions illustrated in Figure 4 are:

(a) Under the pond, $X \geq 0$,

$X = \infty : h_f = h_p$ where h_p = fresh-water head of pond

$X = 0 : h_f = h_o$

(b) Outside the pond, $X \leq 0$,

$X = -\infty \quad h_f = h_a$

$X = 0 \quad h_f = h_o$

It should be noted that if the fluid in the pond is fresh, h_p equals the elevation of the pond level above the reference level. However, if the fluid in the pond is saline and if the saline-fresh water interface occurs below the bottom of the pond (see Figure 3), then the density effects have to be taken into account. In that case, h_p has to be expressed as an equivalent fresh-water head and can be calculated from:

$$h_p = Z_p + \frac{\rho s - \rho f}{\rho_f} (Z_p - Z_i) \qquad (5\text{-}2)$$

where the symbols are the same as for equation 4-16.

The resulting equations for the head in the semi-confined aquifer are:

(a) Under the pond, $0 \leq X \leq \infty$:

$$h_f = (h_o - h_p)\, e^{-X/L} + h_p \qquad (5\text{-}3)$$

(b) Outside the pond, $-\infty \leq X \leq 0$,

$$h_f = (h_o - h_a)\, e^{X/L} + h_a \qquad (5\text{-}4)$$

(c) Beneath the dyke, $X = 0$:

$$h_o = 0.5\, (h_p - h_a) \qquad (5\text{-}5)$$

Equations 5-2 and 5-3 deserve some further examination. Both equations show that the head increases or declines exponentially. In this respect it is important to know that if $X=4L$ in equation 5-2 or $X=-4L$ in equation 5-3, $e^{-X/L}=0.018$. In other words, at a distance 4L from the dyke the head in the aquifer becomes virtually equal to the

head of the pond. Similarly, equation 5-3 shows that if X=-4L the head in the aquifer becomes virtually equal to the head at the top of the aquitard i.e. the pond has very little influence on head in the aquifer beyond X=-4L.

Equations 5-2 and 5-3 have several interesting applications:

1. If h_o is known from an actual measurement and if the head in the semi-confined aquifer at some distance X outside the dyke is also known, L can be determined from equation 5-4. Then, if the transmissivity T of the aquifer is known, the vertical hydraulic resistance of the aquitard can be calculated.
2. f h_o and L are known or estimated, equations 5-3 and 5-4 can be used to predict the influence of a pond on heads and gradi ents in a semi-confined aquifer beneath it.

It is also interesting to note that equations 5-3 and 5-4 indicate that within a distance of L from the dyke 63% of the seepage from the pond into the aquifer occurs and outside the dyke 63% of the upward seepage from the aquifer toward the water table.

Making use of equation 5-4 the total amount of upward seepage outside the dyke area can be calculated. Per unit area, the amount of upward seepage can be expressed as:

$$q = \frac{h_o - h_a}{c} \exp\,(X/L) \qquad (5\text{-}6)$$

which integrated over the region over which the upward seepage occurs, X<0, gives the total seepage Q per unit dyke width:

$$Q = L\,\frac{(h_o - h_a)}{c} \qquad (5\text{-}7)$$

Based on the principle of continuity, it is obvious that the amount Q equals the amount of downward seepage per unit width of dyke within the pond area.

5.3 Circular Basin Model

In Figure 5 the circular basin model is illustrated schematically. Within the pond area water is moving vertically down through the aquitard. In the aquifer itself water is moving horizontally, in a radial pattern away from the centre of the pond. Outside the dyke water is moving vertically upward from the aquifer toward the water table.

Except for the condition that the ratio R/L should be smaller than 2.5, the same assumptions are valid as were made for the straight dyke case.

Because of the complexities related to the solution of this case of groundwater flow, only the final solution will be given here. For a more detailed discussion of this particular flow problem the reader is referred to Verruijt (1970).

For the domain $0 < r < R$, the solution is:

$$h_f = h_p - (h_p - h_o) \frac{I_o (r/L)}{I_o (R/L)} \qquad (5\text{-}8)$$

where I_o = modified Bessel function of zero order and first kind.

For the domain $R < r < \infty$, the solution is:

$$h_f = ha + (h_o - h_a) \frac{K_o (r/L)}{K_o (R/L)} \qquad (5\text{-}9)$$

where K_o = modified Bessel function of zero order and second kind.

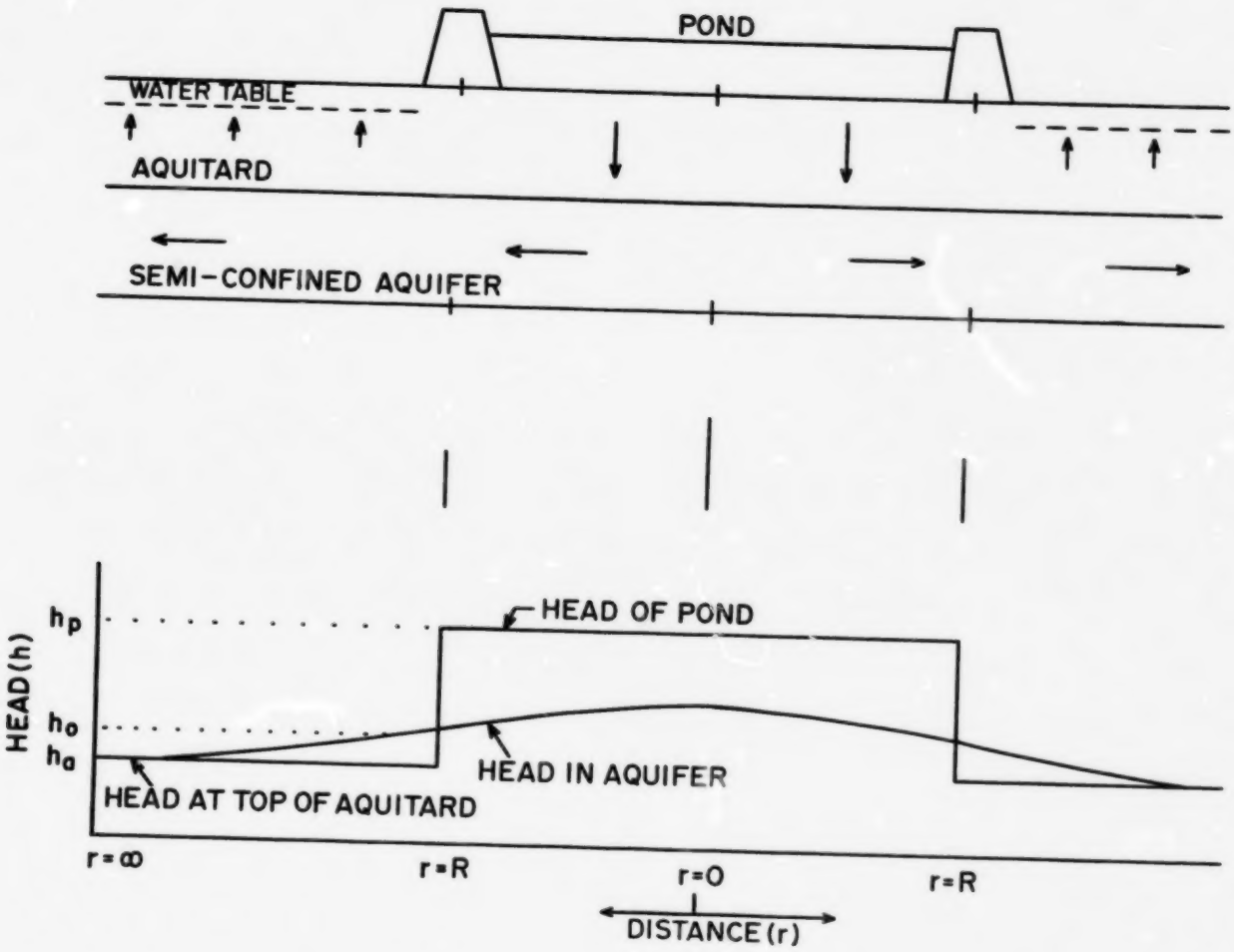

Fig. 5 Schematic illustration of heads across a circular dyke separating a pond from its surroundings.

Equations 5-8 and 5-9 have some interesting implications. Namely, if the ratio R/L is small, for practical purposes R/L $\leq$ 0.1, it follows from equations 5-8 and 5-9 that there is no build-up of head either beneath or outside the pond. In other words, the pond has no significant influence on the water level in the semi-confined aquifer. The ratio R/L will be small if L is large compared to the radius R. This implies that the pond has to be underlain by a thick aquitard and/or an aquifer with a very high transmissivity. These conditions are likely to be quite common for most of the waste disposal ponds in Saskatchewan. If the ratio R/L ranges from 0.1 to 2.5, there will be a head build-up in the semi-confined aquifer beneath and outside the pond. As discussed in the previous section, if R/L > 2.5 the straight dyke case can be applied.

The total volume of downward seepage from a circular pond into the aquifer is given by (Verruijt, 1970):

$$Q = 2\pi R^2 \frac{(h_p - h_a)}{c} K_1(R/L)\ I_1(R/L) \qquad (5\text{-}10)$$

where $K_1(x)$ and $I_1(x)$ are modified Bessel functions.

If R/L becomes small, equation 5-9 reduces to:

$$Q = \pi R^2 \frac{(h_p - h_a)}{c} \qquad (5\text{-}11)$$

which is simply an expression of Darcy's Law and is the maximum amount of downward flow, which occurs if there is no head build-up in the aquifer. The additional terms in equation 5-10 merely indicate that if there is a head build-up in the semi-confined aquifer, the amount of downward flow will be less than the maximum flow.

The Bessel functions I_0, K_0, I and K are tabulated in various handbooks. Verruijt (1970) gives a short table and approximations.

6.0 ANALYSIS OF SETTLEMENT AND EXCESS PORE PRESSURE UNDER A TAILINGS PILE

6.1 Introduction

It has been well established that applying a load upon sediments with a low hydraulic conductivity results in increased pore pressures (i.e. hydraulic heads). If no further loads are applied, the increased pore pressures will cause drainage of the sediments, pore pressures will dissipate, and consolidation will take place. When the load is progressively increased, pore pressure will remain high, but because drainage takes place, there is also ongoing consolidation. For example, despite some interruptions, the salt tailings piles in Saskatchewan represent the case of progressively increasing the load.

Maathuis and van der Kamp (1983) and van der Kamp and Maathuis (1985) provide a case history of the influence of a salt tailings pile on groundwater flow. Based on water level measurements beneath a tailings pile they showed that due to continuous loading high pore pressures were created in a layer of low hydraulic conductivity beneath the pile. The pore pressure in this layer was found to be sufficiently high to prevent downward migration of brine as long as loading continues.

In the next two sections the theory for the analysis of settlement and excess pore pressure will be discussed.

6.2 Analysis of Excess Head Under a Tailings Pile During Continuous Loading

A basic equation in the derivation of the one-dimensional consolidation equation is (Scott, 1980, p. 108):

$$K_v \frac{\partial^2 h}{\partial z^2} = -m_v \frac{\partial \sigma'}{\partial t} \qquad (6\text{-}1)$$

where: K_v = vertical hydraulic conductivity

h = hydraulic head

m_v = coefficient of volume change

σ' = effective stress

In the following derivations it is assumed that K_v', and m_v are constant. This approach allows simple but useful calculations. The actual processes of consolidation may be considerably more complex, involving changes of both K_v' and m_v (see for instance, Tavenas et. al., 1983).

Assuming saturated conditions and highly compressible soil the effective stress σ' is defined by:

$$\sigma' = \sigma - \gamma_w h \qquad (6\text{-}2)$$

where: σ' = effective stress

σ = total stress

γ_w = specific gravity of water

h = hydraulic head

Thus if the total stress is constant

$$\frac{\partial \sigma'}{\partial t} = -\gamma_w \frac{\partial h}{\partial t} \qquad (6\text{-}3)$$

and consequently, equation 6-1 becomes an expression of Terzaghi's equation for one-dimensional consolidation:

$$c_v \frac{\partial^2 h}{\partial z^2} = \frac{\partial h}{\partial t} \qquad (6\text{-}4)$$

where: c_v = coefficient of consolidation

However, under saturated conditions and highly compressible soils, and

However, under saturated conditions and highly compressible soils, and continuous loading, equation 6-2 becomes

$$\frac{\partial \sigma'}{\partial t} = \frac{\partial \sigma_o}{\partial t} - \gamma_w \frac{\partial h}{\partial t} \tag{6-5}$$

where: σ_o = surface load

and the equation for one-dimensional consolidation for continuous loading:

$$c_v \frac{\partial^2 h}{\partial z^2} = \frac{\partial h}{\partial t} - \frac{1}{\gamma_w} \frac{\partial \sigma_o}{\partial t} \tag{6-6}$$

Suppose now that the surface load is increasing at a constant rate R:

$$\frac{\partial \sigma_o}{\partial t} = R \tag{6-7}$$

and that the process has gone on long enough so that the rate of consolidation is in equilibrium with the rate of increase of the load and the excess head is no longer changing. Then:

$$c_v \frac{\partial^2 h}{\partial z^2} = \frac{-R}{\gamma_w} \tag{6-8}$$

Suppose the consolidating layer has a thickness equal to 2d, extends vertically from z=-d to z=d, and that the head in the formation at the bottom and top of the layer is zero (two-way drainage). The boundary conditions are then:

$z = d \qquad h = 0$

$z = -d \qquad h = 0$

and the solution of equation 6-8 is

$$h = -\frac{R}{2 c_v \gamma_w} (z^2 - d^2) \tag{6-9}$$

The maximum excess head occurs at the mid-point of the layer where z=0:

$$h_{max} = \frac{Rd}{2\, c_v\, \gamma_w} \qquad (6\text{-}10)$$

The rate of water drainage out of the layer per unit area is given by:

$$q = 2\, m_v\, d\, R \qquad (6\text{-}11)$$

If the layer drains up and down equally then the drainage each way is q/2. Because highly compressible and fully saturated soils were assumed, equation 6-11 also can be used to calculate the rate of settlement because under these conditions the rate of settlement is equal to the rate of drainage per unit area.

Equations 6-9 and 6-10 have been derived for the case of two-way drainage. However, these equations are also valid for the case of one-way drainage by simply taking the layer thickness as d rather than 2d.

Equation 6-10 has two interesting applications:

1. It shows that measurement of the steady-state value of excess head of the mid-point of the layer allows a determination of the bulk in situ value of the coefficient of consolidation c_v. Consequently, if the coefficient of volume change m_v is known, a bulk vertical hydraulic conductivity can be obtained.
2. Conversely, if c_v is known from other sources or is estimated, the excess head at the mid-point of the layer can be estimated.

Equations 6-9 and 6-10 refer only to the excess head and flow induced by loading. Superimposed on these there may be steady upward or downward flow or other transients.

6.3 Analysis of Settlement and Excess Pore Pressure After Tailings Deposition Ceases

If for some reason loading is suspended for a period of time, an analysis of settlement and excess pore pressure can be carried out.

From consolidation theory it is known that during the latter part of a consolidation process, after the instantaneous load has been applied, the equation for settlement can be written as (Scott, 1980, p. 109):

$$\bar{u} = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(-M^2 T_v) \qquad (6\text{-}12)$$

where: $\bar{u}$ = fraction of the total settlement that has occurred at time t

$M = 1/2\,\pi\,(2m+1)$ for $m = 0, 1, 2, \ldots\ldots$

T_v = time factor = $c_v\, t/d^2$

For values of $T_v > 0.1$ the terms for $m \geq 1$ become very small and equation 6-12 can be written in the form:

$$\log(1-\bar{u}) = 0.0912 - 1.072 \frac{c_v}{d^2}\, t \qquad (6\text{-}13)$$

where: $\bar{u}$ = fraction of the total settlement that has occurred at time t

d = half layer thickness (drainage up and down)

c_v = coefficient of consolidation

On a plot of $\log(1-\bar{u})$ versus t, the latter part of the settlement data should produce a straight line with slope:

$$\Delta \frac{\log(1-\bar{u})}{\Delta t} = 1.072 \frac{c_v}{d^2} \qquad (6\text{-}14)$$

This equation is valid for the analysis of settlement data after a load had been applied instantaneously. However, it can be shown mathematically that as long as the criterion:

$$\frac{c_v t}{d^2} > 0.1 \qquad (6\text{-}15)$$

is satisfied, it is also valid for the analysis of settlement data after non-instantaneous loading. Consequently, without having to know the loading history, settlement data can be analysed and a value for c_v can be determined.

Application of equation 6-14 probably will be limited because of the difficulties in obtaining reliable settlement data. However, it also can be shown mathematically that for the excess pore pressure (or excess hydraulic head h_e) within the consolidating layer an expression similar to equation 6-14 can be derived. If the criterion, expressed in equation 6-15, is fulfilled then:

$$\Delta \frac{\log (h_e)}{\Delta t} = 1.072 \frac{c_v}{d^2} \qquad (6\text{-}16)$$

Thus a semi-log plot of the excess head versus time also allows a determination of c_v. Again, the advantage of this technique for calculating c_v is that the exact times and rates of placement of the load need not to be known.

The bulk in-situ consolidation coefficient thus determined then can be used to calculate the bulk in situ vertical hydraulic conductivity of the consolidating layer provided the coefficient of volume change m_v for that layer is known.

A semi-log plot of the excess head versus time after deposition of tailings ceased also allows estimation of the time which will elapse before hydraulic heads decline to a new equilibrium level. The term new equilibrium level is used here on purpose because the hydraulic head will probably not return to its original level. The reason for this is

that a tailings pile is essentially an artificially created surficial aquifer and consequently, the whole hydrogeological setting has been changed.

In Saskatchewan the in-situ values for the consolidation coefficient range from 3×10^{-7} m^2/sec for clayey silt (van der Kamp and Maathuis, 1985) to 1.0×10^{-6} m^2/sec for thick unfractured till units (van der Kamp et al., 1986) and 4.5×10^{-5} m^2/sec for fractured till (Keller et al., 1986). These values, and the theory presented in this section, indicate that excess heads in typical Saskatchewan settings may be less than 1.0 m and may exceed 100 m.

7. HYDRODYNAMIC CONTAINMENT IN A SEMI-CONFINED AQUIFER

7.1 Introduction

Even if measures, such as installation of liners, are taken to reduce the propensity for vertical migration, ultimately saline water may tend to move slowly downward under a waste disposal basin. The mean velocity of downward movement through the aquitard may be very small. However, due to the possible presence of fractures or sandy zones, the time when saline water enters the underlying aquifer may be unpredictable. In case of fractures, the total inflow of saline water would be small, but if it contains a high concentration of dissolved salts, the resulting contamination of the aquifer could be significant. Once the saline water has reached the aquifer, it will move laterally.

The lateral spreading of the contaminated water within the semi-confined aquifer can be controlled by means of hydrodynamic containment. Essentially this control method reverses the hydraulic gradient within the aquifer by means of pumping from one or more control wells, installed down-gradient from the waste disposal area.

7.2 Theory of Hydrodynamic Containment in a Semi-Confined Aquifer

Almost any naturally occurring aquifer is non-uniform and discontinuous. However the theory of flow in a uniform, infinite aquifer can be used to obtain a first approximation of the flow patterns in an aquifer when a control well is pumped. Detailed analysis of the flow pattern requires numerical modelling, and a great deal of data. The simple analysis, combined with fine-tuning of the control system once it is operating, will probably suffice for practical purposes.

Before pumping starts, the flow in the aquifer is steady. The pre-existing or background flow includes the flow due to the waste disposal basin. When pumping for hydrodynamic control starts, a "drawdown zone" spreads out from the well(s). Eventually this drawdown zone stabilizes and it is this stabilized condition that is of most interest with respect to the feasibility of hydrodynamic control.

The new flow pattern in the aquifer then consists of a simple superposition (vector addition) of the background flow plus the radial flow to the well(s).

For the simple analysis, it may be assumed that the background flow is unidirectional with a hydraulic gradient equal to G_o, i.e., the background head distribution is given by:

$$h_o = G_o\, x + H_o \qquad (7\text{-}1)$$

where: x = distance up-gradient from the well

H_o = initial head at x=o

For a single well pumping from a semi-confined aquifer, the steady-state drawdown is given by (see equation 3-4):

$$s = \frac{Q}{2\pi T} K_o\left(\frac{r}{L}\right) \qquad (7\text{-}2)$$

where: s = drawdown

Q = pumping rate

T = transmissivity of the aquifer

r = radial distance from the well

L = leakage factor

The function K_o is the modified Bessel function of order zero and second kind as tabulated in many handbooks.

The leakage factor L is defined by the equation:

$$L = (T\ b\ /K_v)^{1/2} \qquad (7\text{-}3)$$

where: b = thickness of the overlying aquitard

K_v = vertical hydraulic conductivity of the overlying aquitard

For this simple model of a single well pumping from an infinite semi-confined aquifer with unidirectional background flow, the total head is given by:

$$h = G_o x + H_o - \frac{Q}{2\pi T} K_o \left(\frac{r}{L}\right) \qquad (7\text{-}4)$$

This equation allows a straightforward calculation of the head and the hydraulic gradient anywhere in the aquifer. The Bessel function (K_o (r/L)) decreases rapidly with increasing value of r/L. At distances more than 4L away from the well, the flow in the aquifer is practically unaffected by the pumping from the control well.

The distance, x_s, downstream from the well to the "stagnation point" is determined easily and is a useful parameter for design purposes. At the stagnation point the hydraulic gradient is zero, all water directly up-gradient from the point moves towards the control well; all water down-gradient from the point moves down and eventually "escapes". The distance, x_s, can be determined from the indirect equation.

$$K_1\left(\frac{x_s}{L}\right) = \frac{2\pi T\ L\ G_o}{Q} \qquad (7\text{-}5)$$

where K_1 is the modified Bessel function of the second kind and order one as tabulated in handbooks.

As can be seen from the equations presented, the important aquifer parameters with respect to the design of a hydrodynamic control system are the transmissivity T, the leakage factor L and the background gradient G_o. As an example, assume the parameters in equation 7-4 have the following values:

$$Q = 1.0 \times 10^{-3} \text{ m}^3/\text{sec.}$$

$$T = 1.0 \times 10^{-4} \text{ m}^2/\text{sec.}$$

$$L = 1000 \text{ m}$$

$$G = 1.5 \times 10^{-3} \text{ (dimensionless)}$$

For a pumping well located 500 metres down gradient of a waste disposal basin, Figure 6 depicts the resulting steady-state distribution of the hydraulic head and flowpaths in a semi-confined aquifer with a uniform background flow. An actual situation will probably be more complex, however, Figure 6 shows the essential elements of the flow pattern associated with hydrodynamic control. Although the influence of the transmissivity and background gradient are important, the leakage factor L is the most important parameter in the design of a hydrodynamic containment system. Figure 6 shows that if the width of the waste disposal basin is equal to the leakage factor, one pumping well would be sufficient to achieve containment. However, when the width of the basin is much greater than L, extreme care has to be taken in the design of a hydrodynamic containment system. In that case more than one well will be required and also the location of the wells becomes very important. If wells are wrongly located, conditions could be created whereby migration of saline fluid not only will continue but locally will continue at an increased rate (Keely et al, 1983).

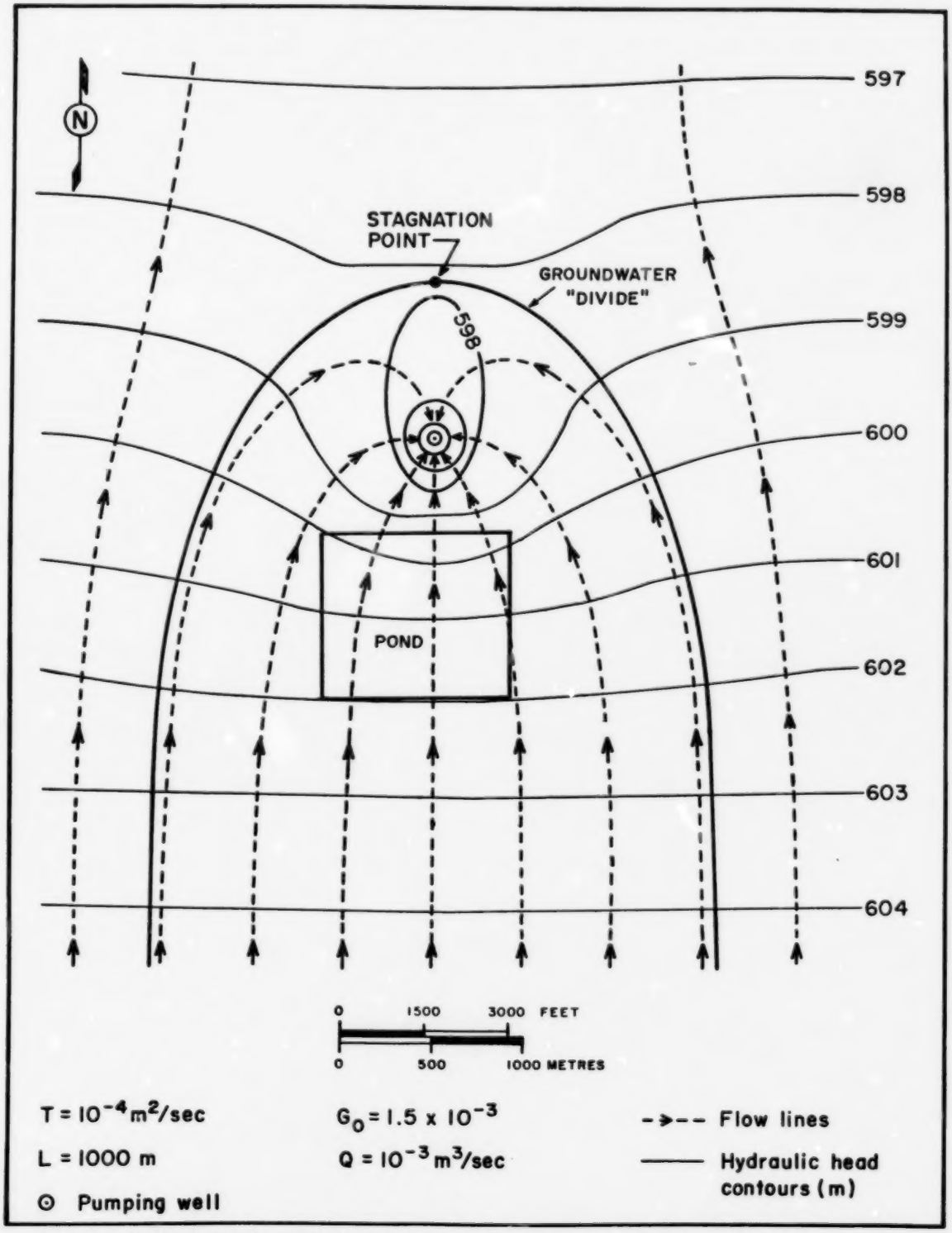

Fig. 6 Hydrodynamic containment in a uniform semi-confined aquifer with a single control well.

It should be noted that once steady-state conditions are reached, all the water pumped from the semi-confined aquifer is replaced by vertical downward inflow from the unconfined surficial aquifer. There is no net removal of water from the semi-confined aquifer, and the flow in this aquifer far down-gradient from the well is equal to the original background flow.

Another important aspect of hydrodynamic containment in a semi-confined aquifer is that each control well will exert a strong influence over gradients and flow within the aquifer in all directions, to a radius of about L. For example, as illustrated in Figure 6, in the area just down-gradient of the stagnation point the hydraulic head in the aquifer has been significantly lowered. In this area the vertical downward flow from the surficial aquifer has been increased while the flow in the semi-confined aquifer is away from the stagnation point. If for example, the surficial aquifer also contains saline water, some contamination could still migrate down-gradient. Also, because hydrodynamic containment results in lowering of the hydraulic head beneath the waste disposal basin, there will be an increased downward flow from the pond into the semi-confined aquifer.

The above analysis is based on the assumption of a single fluid with constant density. If there is a "tongue" of saline water with markedly different density at the bottom of the aquifer, the situation becomes much more complex. The hydraulic "trap" set up by pumping of the containment wells must then be strong enough to overcome the extra head of the saline fluid at the deepest portions of the aquifer (see section 4).

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